Summary

1. **Discrete R.V. v.s. Continuous R.V.**

integers or discrete points (counted) — any value on a continuous range (measured)

(a) Identify the given random variable as being **discrete** or **continuous**:

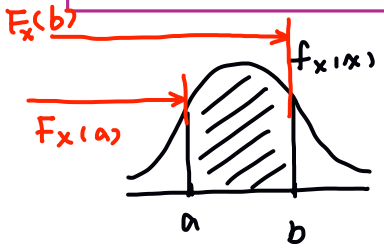
- D • The number of people now driving a car in the United States.
- C • The amount of rainfall in a given month
- C • The running time of a randomly selected movie
- D • The number of eggs a hen lays in a day
- C • The weight of the gold stored in Fort Knox

(b) Probability **Mass** Function v.s. Probability **Density** Function

Discrete pmf: $P_X(x) = P(X=x)$ ← it's a probability (mass at a point)

Continuous pdf: $f_X(x)$ ← it's not a probability; it's what you integrate to get probability. (mass is spread over an interval)

$$P(a \le X \le b) = \int_a^b f_X(x)\,dx$$



Area ⇔ probability

$$P(X=a) = \int_a^a f_X(x)\,dx = 0$$

In continuous case, we care about intervals of X, not the individual values of x.

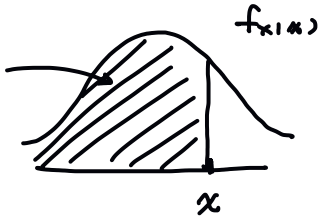
$P(a \le X \le b) = P(a < X < b)$

To be valid:

(c) Cumulative Distribution Function (**CDF**)

$\sum_x P_X(x) = 1$ Discrete CDF: $F_X(x) = P(X \le x) = \sum_{t \le x} P_X(t)$

$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$ Continuous CDF: $F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(t)\,dt$



$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$$

$$\frac{d}{dx} F_X(x) = f_X(x)$$

$$P(a \le X \le b) = F_X(b) - F_X(a)$$

(d) Expected Value & Variance

Discrete

$E(X) = \sum x P_X(x)$

$E(g(X)) = \sum g(x) P_X(x)$

Continuous

$$E(X) = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx$$

$$E(g(X)) = \int_{-\infty}^{\infty} g(x)\, f_X(x)\,dx$$

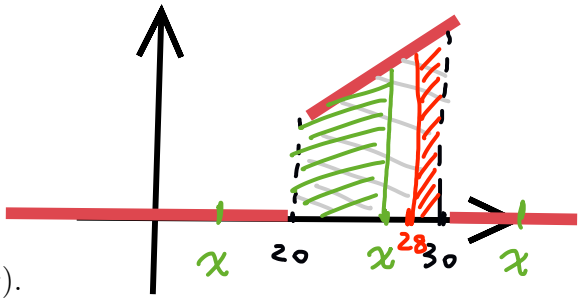
$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$\mathrm{Var}(X) = E(X^2) - [E(X)]^2$$

Example 2. Consider the following PDF: $f_X(x) = \frac{x}{250}$ for $20 \le x \le 30$; $f_X(x) = 0$ otherwise.

(a) Verify that $f_X(x)$ represents a probability density function.

$$\int_{20}^{30} \frac{x}{250}\,dx = \frac{1}{500}x^2\Big|_{20}^{30} = \frac{1}{500}(900-400) = 1 \quad \checkmark$$



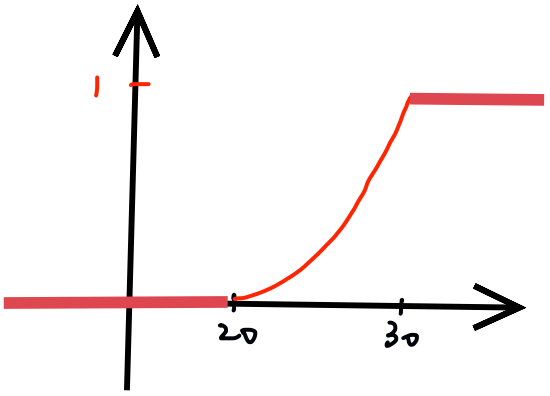
(b) Determine the cumulative distribution function, $F_X(x)$.

$$F_X(x) = P(X \le x) = \int_{20}^{x} \frac{t}{250}\,dt$$

$$= \frac{1}{500}t^2\Big|_{20}^{x}$$

$$= \frac{1}{500}(x^2 - 400)$$

$$F_X(x) = \begin{cases} 0 & x \le 20 \\ \frac{1}{500}(x^2-400) & 20 \le x \le 30 \\ 1 & x \ge 30 \end{cases}$$



as $x \to -\infty$, $F_X(x) \to 0$

as $x \to \infty$, $F_X(x) \to 1$

(c) Find $P(X > 28)$.

$$P(X > 28) = \int_{28}^{30} \frac{x}{250}\,dx$$

$$\overset{OR}{=} 1 - F_X(28) = \boxed{\frac{116}{500}}$$

(d) Find μ_X.

$$E(X) = \int_{20}^{30} x \cdot \frac{x}{250}\,dx = \boxed{\frac{76}{3}}$$